“Quantum Mechanics”
As the Mechanics of the time region

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The preliminary results of a critical study of the Wave Mechanics carried out in the light of the knowledge of the Reciprocal System of theory have been reported earlier.[1] Some of its important findings are as follows. While the Wave Mechanics has been very successful mathematically, it contains some fundamental errors. The principal stumbling block has been the ignorance of the existence of the time region and its peculiar characteristics. The crucial points that need to be recognized are that the wave associated with a moving particle, in a system of atomic dimensions, exists in the *equivalent space* of the time region; and that switching from the particle view to the wave view is equal in significance to shifting from the standpoint of the three-dimensional spatial reference frame to that of the three-dimensional temporal reference frame that is germane to the time region. To imagine that even gross objects have a wave associated with them is a mistake: the question of the wave does not arise unless the phenomena concerned enter the time region.

One corollary is that the theorists’ assumption that the wave associated with the moving particle is spatially co-extensive with the particle is wrong since the former exists in the *equivalent space*, not in the extension space of the conventional spatial reference system. The Uncertainty Principle stems from the theorists’ practice of resorting to wave packets.

It has further been shown that the probability connotation of the wave function arises from the two facts that the wave is existent in the three-dimensional temporal manifold, and that locations in the three-dimensional temporal manifold are only randomly connected to locations in the three-dimensional spatial manifold. The *non-local* nature of the forces (motions) in the time region also follows from these facts.

Calculations based on the inter-regional ratios applicable confirm Larson’s assertion that the measured size of the atom is in the femtometer range and hence what is found from the scattering experiments is the size of the atom itself—not of a nucleus.

From the above study it became abundantly clear that the critics’ comments that the small-scale world is not intrinsically rational and that the Quantum theory cannot be understood intuitively were wrongly founded. What was really missing was the knowledge of the existence and characteristics of the time region, the region inside the natural unit of space, where only motion in time is possible. Since our knowledge of the Reciprocal System helped straighten some of the conceptual kinks of the Wave Mechanics and has indicated that its original basis has been rightly (though unconsciously) founded, an attempt has been made to inquire into its mathematical aspects in order to see whether they are valid in the light of our understanding of the Reciprocal System. The results of this inquiry are reported in this article.

1 K.V.K. Nehru, “The Wave Mechanics in the Light of the Reciprocal System,” *Reciprocity*, Vol. XXII, No. 2, Autumn 1993, p. 8–13.

1 Where Do We Stand

Before proceeding further it would be desirable to take a stock of the atomic situation from the point of view of the Reciprocal System.

Firstly, Larson[2] asserts that the atom is without parts, that it is a unit of compound motion, motion being the basic constituent of the physical universe. This means that both the nucleus and the so-called orbital electrons are non-existent.

Secondly, he argues that there is no electrical force either, involved in the atomic structure. This, therefore, leaves gravitation and the space-time progression as the only two motions (forces) that operate inside the time region with, of course, the appropriate modifications peculiar to the time region introduced into them.

Under these circumstances the question of a "nuclear" force does not arise at all. But it is perfectly legitimate to inquire what forces (motions) are encountered by a particle as it approaches the vicinity of an atom, and indeed, as it enters the very atom itself. Equally important is to inquire into the mechanics of the converse process of the emission of a particle by the atom.

2 The Wave Equation

The most fundamental starting point for the mathematical treatment in the Quantum Mechanics is the wave equation. The wave equations in the quantum theory govern the wave functions associated with the particles, and correspond to Newton's laws of classical mechanics. From our earlier study we have seen that changing from the particle picture to the wave picture is a legitimate strategy that needs to be adopted on entering the time region, as it is tantamount to shifting from the conventional three-dimensional spatial reference frame of the time-space region to the three-dimensional temporal reference frame of the time region. Therefore the next logical step is to examine how the governing equations of the wave phenomena have been arrived at and see if it is in consonance with the Reciprocal System.

Since it is always possible to constitute a wave of any shape by superposing different sinusoidal waves of appropriate wavelengths and frequencies, we shall limit our discussion to these elementary sinusoidal waves. The relation between the wave number k and the wavelength λ on the one hand, and that between the angular frequency ω and frequency ν on the other, are as follows:

$$k=\frac{2\pi}{\lambda};\ \omega=2\pi\nu \tag{1}$$

The wave speed u is given by

$$u=\lambda\cdot\nu=\frac{\omega}{k} \tag{2}$$

The general functional forms of sinusoidal waves are

$$\left.\begin{matrix}\sin(kx\pm\omega t)\\ \cos(kx\pm\omega t)\end{matrix}\right\} \tag{3}$$

2 Larson, Dewey B., *The Case Against the Nuclear Atom*, North Pacific Publishers, Oregon, USA, 1963.

and in complex exponential form (see Appendix I: Euler's Relations)

$$e^{i(kx \pm \omega t)} \tag{4}$$

where the imaginary unit i is defined by $i^2 = -1$.

Complex functions involve a *real* part and an *imaginary* part. Since at this stage of our discussion the nature of the wave function of particles is yet unknown, there is no theoretical reason to exclude complex functions. Let us bear in mind that the criterion of judgment is what is possible in the time region, not what is possible in the time-space region. To be sure, observable quantities in the time-space region ought to be real. However, by virtue of the second power relation between corresponding quantities in the time region and the time-space region, the observable value of a time region quantity would still be real even if it were to be imaginary in the time region (e.g. a quantity $i \cdot \nu$ in the time region would appear as $(i \cdot \nu)^2$, that is, $-\nu^2$ in the outside region).

2.1 Radiation Waves

Let us derive the governing equation for the wave propagating at constant speed, like that of radiation. First we note the relation between the momentum p of the wave and the wave number k, and the energy E and its angular frequency ω,

$$p = \hbar k \; ; E = \hbar \omega \tag{5}$$

where $\hbar$ is Planck's constant h divided by 2π.

From the energy-momentum relationship of the wave, $p^2c^2 = E^2$, (c being the constant wave speed) we have

$$\begin{gathered} p^2 = \frac{1}{c^2} E^2 , \\ \hbar^2 k^2 = \frac{1}{c^2} \hbar^2 \omega^2 , \\ k^2 = \frac{1}{c^2} \omega^2 \end{gathered} \tag{6}$$

Assuming the simplest wave form, that of a sine wave, we write the wave function in complex exponential form as

$$\Psi(x, t) = A \cdot e^{i(kx - \omega t)} \tag{7}$$

where A is an arbitrary constant. For such a function,

$$\left. \begin{aligned} \frac{\partial \Psi}{\partial x} &= i k \cdot \Psi \\ \frac{\partial \Psi}{\partial t} &= -i \omega \cdot \Psi \end{aligned} \right\} \tag{8}$$

That is, taking the derivative with respect to x is equivalent to multiplying by ik, and taking the derivative with respect to time t is equivalent to multiplying by $-i\omega$. Thus

$$\left.\begin{aligned}\frac{\partial^2\Psi}{\partial x^2}&=(i\,k)^2\cdot\Psi=-k^2\cdot\Psi\\ \frac{\partial^2\Psi}{\partial t^2}&=(-i\,\omega)^2\cdot\Psi=-\omega^2\cdot\Psi\end{aligned}\right\} \tag{9}$$

Substituting these in the last of Equation (6) we obtain

$$\frac{\partial^2\Psi}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} \tag{10}$$

which is exactly the wave equation we are seeking (see Appendix II: The General Equation of a Constant Speed Wave).

2.2 Matter Waves

At the instance of his mentor Peter Debye, Erwin Schrödinger made a detailed study of the wave hypothesis advocated in 1924 by de Broglie. Schrödinger noted that the energy-momentum relationship of a free particle (not acted by forces) of mass *m*

$$\frac{p^2}{2m}=E \tag{11}$$

leads to the wave number–angular frequency relation

$$\frac{\hbar^2 k^2}{2m}=\hbar\cdot\omega \tag{12}$$

From Equations (2) and (12) we see that the wave speed in this case is given by

$$u=\frac{\hbar k}{2m} \tag{13}$$

Therefore the speed of the matter waves is not constant like that of the radiation waves, but is a function of the wave number *k*. Equation (12) could be rearranged as

$$-\frac{\hbar^2}{2m}(i\,k)^2=i\,\hbar(-i\,\omega)$$

Multiplying both sides by Ψ, we can at once see from Equations (8) and (9) that

$$\frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}=i\,\hbar\frac{\partial\Psi}{\partial t} \tag{14}$$

which is the governing equation for the wave associated with the free particle that we are looking for. This is the Schrödinger equation for the free particle. It is the equation in the time region which corresponds to Newton's first law of the time-space region.

In order to include *interactions* of the particles with the environment we note that the total energy of such a particle consists of the kinetic energy and the potential energy. The latter could be taken to be dependent only on position and represented by a potential energy function *V(x)*. Thus for a conservative system we have the constant total energy *E* given by

$$\frac{p^2}{2m}+V(x)=E \tag{15}$$

The corresponding wave number-frequency relation, associating frequency with the total energy, is

$$\frac{\hbar^2 k^2}{2m}+V=\hbar\omega$$

Adopting Equations (8) and (9) as before, we arrive at the Schrödinger wave equation with interaction present

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}+V(x)\cdot\Psi=i\hbar\frac{\partial\Psi}{\partial t} \tag{16}$$

This corresponds in the time region to Newton's second law in the time-space region.

As can be seen from the foregoing derivations, nothing against the principles of the Reciprocal System has been introduced so far. Hence the Schrödinger equations can be admitted as legitimate governing principles for arriving at the possible wave functions of an hypothetical particle of mass m traversing the time region, with or without potential energy functions as the case may be. We may note in the passing that often considerable mathematical dexterity is required in solving these differential equations, though computer-oriented numerical methods are fast replacing closed-form solutions.

Any wave corresponding to a state of definite energy E has a definite frequency $\omega = E/\hbar$. Therefore from Equation (7) we can write

$$\Psi(x,t)=A\cdot e^{\frac{-iEt}{\hbar}}\cdot\psi(x) \tag{17}$$

where $\psi(x)$ is a function of space variable only. Inserting the above into Equation (16) and dividing out the factor $e^{-iEt/\hbar}$ throughout, we get the differential equation to be satisfied by $\psi(x)$

$$-\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2\psi}{\partial x^2}+V(x)\cdot\psi(x)=E\cdot\psi(x) \tag{18}$$

which is referred to as the *time-independent* Schrödinger equation. This equation is less general and is valid only for states of definite total energy.

3 States of Negative Energy

It is instructive to see what the solutions of Schrödinger equation turn out to be. Firstly, in any region of constant potential energy V, we see that the solution of Equation (18) is a sinusoidal function,

$$\left.\begin{array}{c}\psi(x)=A\cdot\sin(kx)\quad or\quad A\cdot\cos(kx)\\ k^2=\dfrac{2m\cdot(E-V)}{\hbar^2}\end{array}\right\} \tag{19}$$

(E - V) being the kinetic energy.

3.1 The Step Function

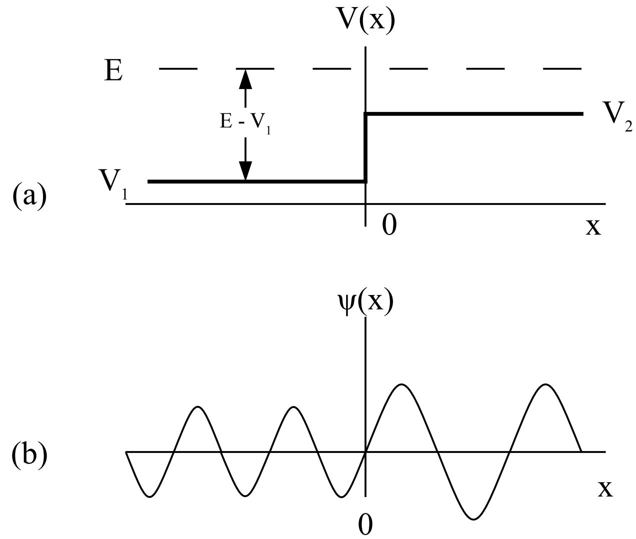

Figure 1: Potential Energy Step

In Figure 1(a) we picture a step-function potential energy, which is constant at V_1 and V_2 respectively in two different regions. A possible wave function corresponding to this case is shown in Figure 1(b). The particle's greater kinetic energy (E - V_1) in the region $x < 0$ is reflected in its larger wave number (smaller wavelength) in this region. Also since its speed in this region is greater, it spends comparatively less time in this region, and this reflects as its smaller amplitude in this region.

An interesting case occurs when the potential energy V in any region is greater than the total energy E. Here the kinetic energy, E - V, becomes *negative!* This is physically impossible in the time-space region and the particle can never enter such region. However, the situation is different in the time region: Equation (18) has valid solutions in the region, with k from Equation (19) taking on *imaginary* values,

$$\left.\begin{aligned}\psi(x) &= A\cdot e^{\pm b x}\\ b &= i\cdot k\end{aligned}\right\} \tag{20}$$

The sign of the exponent is so chosen as to see that ψ tends to zero for large x. Figure 2 illustrates this case: in the region $x > 0$ we see that E is less than the potential energy. The wave function is sinusoidal in the region of positive kinetic energy and is exponential in the region of negative kinetic energy. Both functions join smoothly at $x = 0$ with a first order continuity. The penetration of the wave function into the region of negative kinetic energy has no classical analog and is purely a phenomenon of the time region.

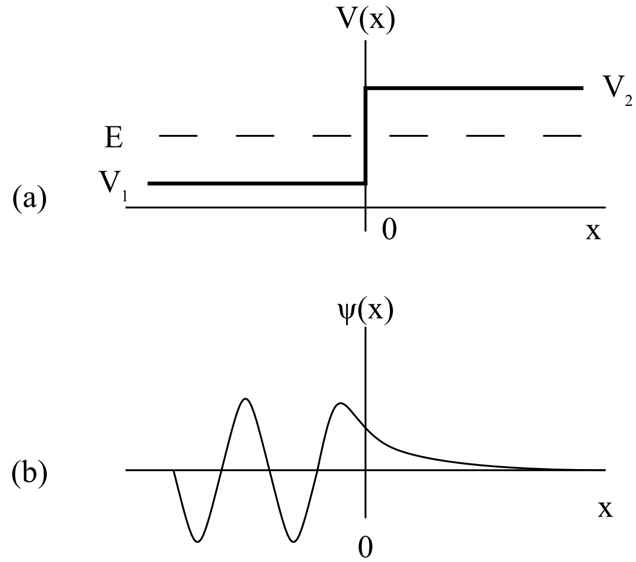

Figure 2: Negative Kinetic Energy

3.2 Explanation of the Negative Energy States

When we turn to the Reciprocal System for an explanation of the possibility of the existence of negative energy states, what we find is as follows. In the time-space region, that is, in the context of the three-dimensional spatial reference frame, speed (space/time) is vectorial, that is, can have direction in space and therefore could take on positive or negative values. This is because in this case space is three-dimensional and time is scalar. In this frame, energy, which is one-dimensional inverse speed (time/space), is scalar, and can take on zero or positive values only. On the other hand, the time region is a domain of the three-dimensional temporal reference frame. In this case time is three-dimensional and space is scalar. Consequently the inverse speed (namely, energy) is the quantity that is "directional," that is, can take on a "temporal direction" in the context of the three-dimensional temporal reference frame. Therefore it is perfectly possible for it to take on negative values as well. (It must be cautioned that "direction in time" has nothing to do with direction in space; it is to be understood that we are only speaking metaphorically.) Further, in the time region, speed is the quantity that is scalar, an example being the net total speed displacement of the atom, namely, the atomic number *Z*.

Moreover the possibility that even potential energy (being an inverse speed) could be "directional" in the three-dimensional time, and hence be represented by complex numbers in the time region, cannot be overlooked. Indeed the Quantum theorists find it necessary to adopt the complex potential $V+i\cdot W$ in place of V in scattering theory. Here the wave number k becomes complex and is written as $k+i\cdot q\cdot b$ of Equation (20) becomes $b = i(k + i\cdot q) = -q + i\cdot k$, and we have

$$\psi=\left(A\cdot e^{-q x}\right)\left(e^{i k x}\right) \tag{21}$$

We can at once see that this is the wave function of a traveling wave of whose amplitude decreases as it advances, and therefore represents a beam of particles some of which are getting absorbed.

3.3 The Potential Energy Barrier

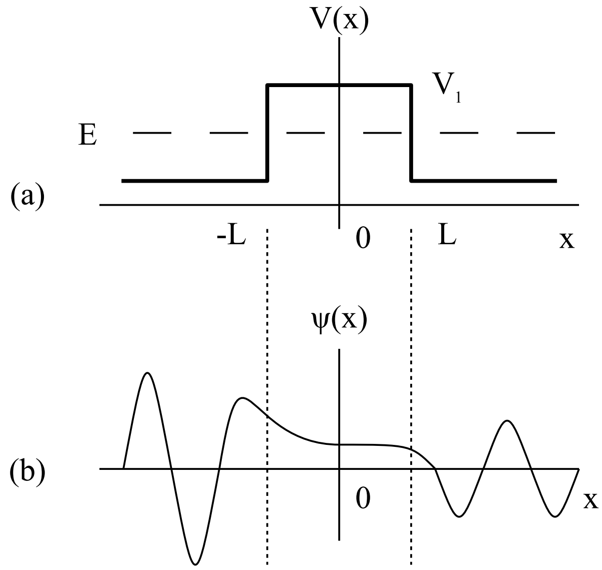

Figure 3: Potential Energy Barrier

An interesting situation arises when two regions of positive kinetic energy occur separated by a *potential energy barrier* that is higher than the total energy as shown in Figure 3(a). In the central region (of negative kinetic energy) the wave function is exponential, while it is sinusoidal on either side as shown in Figure 3(b). At either boundary the function and its first derivative are continuous. From this it is apparent that the particle represented by the wave has a non-zero probability of appearing on the other side of the barrier! While this is a real time region phenomenon that has been observed (the “tunneling”), it has no analog in the time-space region (classical mechanics).

3.4 The Potential Energy Well

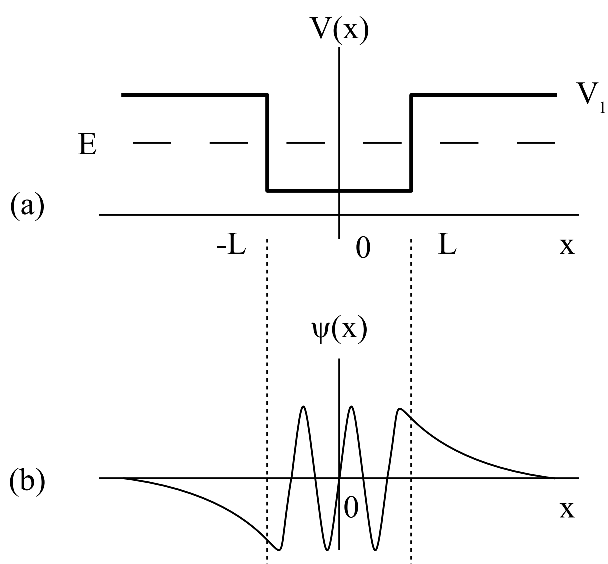

Figure 4: Potential Energy Well

The last case of interest we wish to consider is that of a potential w*ell* as shown in Figure 4(a), wherein the total energy E is less than the potential energy V_1 in the outer regions. As before, we find that the wave function is sinusoidal in the (central) region of positive kinetic energy, and is exponential in the (outer) regions of negative kinetic energy, maintaining first order continuity at the boundaries. But here a new factor emerges, namely, that if we choose an arbitrary value of E, it might become necessary to adopt growing exponentials in the outer regions (for example, e^{+bx} for $x > L$) so as to satisfy the continuity conditions at the boundary. This therefore leads to an unreal state of affairs. The physical requirement is that the wave function goes towards zero with increasing space coordinate in the outer regions. This necessitates the choice of shrinking exponentials in the outer regions (for example, e^{-bx} for $x > L$). This requirement, coupled with the continuity constraints at the boundary, limits the possible energies to a series of distinct levels, each with its own wave function. Thus, well-type potential energy functions give rise to set of possible discrete energy levels. This fact can be seen directly to lead to the explanation of several observable facts including the atomic spectra.

4 Origin of the Pauli Exclusion Principle

The so-called *exclusion principle* was originally promulgated by Wolfgang Pauli. This is an empirical law to which no exception was ever found. It has been a heuristic guiding rule for understanding many an important quantum phenomenon. In spite of its important role, the explanation of its origin has defied the theorists. Therefore that this explanation is now forthcoming from the Reciprocal System is a point in favor of the general nature of the latter theory.

4.1 The Spin

But first we must recognize a point that we have been emphasizing,[3,4] namely, that rotational space is as fundamental as the linear (extension) space. Larson explains: "…the electron is essentially nothing more than a rotating unit of space. This is a concept that is rather difficult for most of us when it is first encountered, because it conflicts with the idea of the nature of space that we have gained from a long-continued, but uncritical, examination of our surroundings. …the finding that the 'space' of our ordinary experience, extension space, as we are calling in this work, is merely one manifestation of space in general opens the door to an understanding of many aspects of the physical universe…"[5] He points out that an atom, for example, can exist in a unit of rotational space as it can in a unit of extension space.

In a paper entitled "*Photon as Birotation*"[6] we have derived that the basic unit of angular momentum is ½$\hbar$. Now we find that the Quantum theorists have been referring to this basic unit of rotational space as the *spin.* In addition to the three space coordinates, spin is treated as a fourth coordinate. Thus two different particles can occupy the same location in extension space at the same time if their spin coordinate differs.

4.2 Indistinguishability

In connection with a class of elementary particles, we know that any two individual particles (say, two electrons) are absolutely alike. In the time-space region, the fact that two particles are identical presents

3 Nehru K.V.K., "The Law of Conservation of Direction," *Reciprocity*, Vol. XVIII, № 3, Autumn 1989, p. 3.
4 Nehru K.V.K., "On the Nature of Rotation and Birotation," *Reciprocity*, Vol. XX, № 1, Spring 1991, p. 8.
5 Larson D.B., *Basic Properties of Matter*, International Society of Unified Science, Utah, USA, 1988, pp. 102-3.
6 Nehru K.V.K., "The Photon as Birotation," *Reciprocity*, Vol. XXV, № 3, Winter 1996-97, pp. 11–16

no complications since they can be kept distinguished by their respective locations. But in the quantum phenomena, because of the *non-local* nature of the time region, no such distinction is possible. This intrinsic indistinguishability gives rise to some special constraints. Let us take $\psi(1,2)$ to be the wave function of two indistinguishable particles with particle 1 at location r_1 (whose coordinates include the spin coordinate also) and particle 2 at location r_2. Then $[\psi(1,2)]^2$ represents the probability distribution for particle 1 to be at r_1 and particle 2 to be at r_2. Since we cannot distinguish between the particles, the wave function should be of such a form that it results in the same probability distribution if we interchange the two particles in ψ. That is

$$[\psi(1,2)]^2 = [\psi(2,1)]^2$$

This can be satisfied in two ways,

$$\left.\begin{aligned} \psi(1,2) &= +\psi(2,1) \\ \psi(1,2) &= -\psi(2,1) \end{aligned}\right\} \tag{22}$$

The first type of wave functions are referred to as the *symmetric* and the second as the *antisymmetric* functions.

Now the empirical finding is that the wave functions of particles like protons and neutrons which are known to have half-integral spin ($\frac{1}{2}\hbar$) are antisymmetrical, and those of particles with integral spin (like the photons) are symmetrical. The most fundamental statement of Pauli exclusion principle goes somewhat like this: “Any permissible wave function for a system of spin-½ particles must be antisymmetric with respect to interchanging of all coordinates (space and spin) of any pair of particles.” But enunciating a principle is quite different from explaining its origin, and the fact is that no theoretical explanation has been found for this empirical finding. One author writes: “For reasons that are not clearly understood, for electrons, protons, neutrons, and all other spin-½ particles, the *minus* sign is chosen…”[7]

4.3 The Two Types of Reference Points

From the Reciprocal System we have now the explanation. Let us recall that in the universe of motion there are two types of reference frames—the conventional, stationary three-dimensional spatial reference frame (or its cosmic analog, the three-dimensional temporal reference frame) and the moving natural reference frame. We also have two kinds of objects, those having independent motion like the gravitating particles and those having no independent motion of their own and hence are stationary in the natural reference frame, like the photons and those particles having *potential mass*[8] only. The *reference point* for the scalar inward motion of the gravitating particle is the particle itself. Thus if there are two locations *A* and *B* in the three-dimensional reference frame with this particle situated at *A*, say, its gravitational motions appears in the direction *BA*, because it is inward, toward itself. If now the particle is shifted to location *B*, the direction of its gravitational motion seems reversed, being in the direction *AB*. This is the origin of the antisymmetry of the wave functions of such particles.

As already remarked, a unit of one-dimensional rotation carries unit spin ($\frac{1}{2}\hbar$). The resultant spin of a two-dimensional rotation with unit spin in each dimension is 1×1 = 1 (that is, $\frac{1}{2}\hbar$) or is 1×(-1) = -1 (that is, $-\frac{1}{2}\hbar$). On the other hand, the resultant spin of a birotation (like the photon) is 1+1 = 2 (that is, $\hbar$) or 1-1 = 0. Since gravitation arises out of the two-dimensional rotation, we can see that a gravitating particle carries spin-½. Thus the wave function of spin-½ particles turns out to be antisymmetric.

7 Cohen, B. L., *Concepts of Nuclear Physics*, Tata McGraw Hill, India, 1971, p. 38.

8 Larson D. B., *Nothing But Motion*, North Pacific Publishers, Oregon, USA, 1979, pp. 141-2, 165-7.

On the other hand, the reference point for the motion of particles like the photons is the location in the natural reference frame, or what Larson calls the "absolute location." The natural reference frame is not a spatial manifold; nor is it a temporal manifold. It is a speed manifold: each location in it is moving at unit speed, one unit of space per unit of time. Suppose that the spatial separation between two locations in this frame (the absolute locations) increases by n natural units of space. Because of the unit speed criterion, there is concomitant increase in the separation in time by n natural units of time, making $n/n = 1$. The expansion in space is completely nullified by the expansion in time (because an increase in space is equivalent to a decrease in time and vice versa), and from a *space-time* point of view there is no separation between absolute locations.

In the context of the three-dimensional reference frame, photons appear to move *outward* from the point of their origin. But we have already seen that the photon is stationary in the absolute location. Its apparent motion is the outward motion of the absolute location (in which it is situated) away from all other absolute locations. The crucial point that should now be recognized is that *outward from one absolute location is still outward from any other absolute location* because of the equivalence of these absolute locations as explained above. Therefore, interchanging the location of the photon between two such absolute locations has no effect on the sign of its wave function. That is, the wave function of such particles is symmetric. One final word is in order: all that has been said above is also true in the time region, except that the scalar direction *outward* in the time-space region manifests as *inward* in the time region and vice versa.

5 Potentials in the time region

Finally it might be of interest to explore the nature and type of the potential energy functions V (see Equation (15)), in the time region. In view of the maiden nature of the investigation and the insufficient time available, the results reported in this section may have to be treated as tentative.

5.1 Dimensional Relations across the Regions

Discussing the effect of the inversion of space and time at the unit level on the dimensions of inter-regional relations, Larson[9] shows that the expressions for speed and quantities related to speed in the time region are the second power expressions of the corresponding quantities belonging to the time-space region. This is because motion (speed) has a spatial component and a temporal component. Since unit space is the minimum that can exist, within the time region—the region *inside* unit space—the spatial component of a speed remains constant at 1 unit and all variability can be in the temporal component, t, only. By virtue of the reciprocal relation between space and time the t units of time are equivalent to $1/t$ unit of space and manifest so in the time region. That is why Larson uses the term *equivalent space* (that is, inverse space) as synonym for time region. The *equivalent speed* in the time region is, therefore, given by the ratio of the equivalent space to time, $(1/t)/t = 1/t^2$. This quantity is the second power expression of the speed in the time-space region with 1 unit of space component and t units of time component, namely, $1/t$.

In an earlier article[1] we have identified two different zones of the time region, namely, the one-dimensional and the three-dimensional. The second power relation mentioned above could be seen to apply specifically to the one-dimensional zone, the zone of *one-dimensional rotation* associated with the atoms or subatoms. On the other hand, for the three-dimensional zone—where the compound

9 *Ibid*., p. 155.

motions constituting an atom exist—the situation is different because the basic rotation that constitutes the atom is two-dimensional. The temporal component of a two-dimensional rotation in the time region would be t^2, and its spatial equivalent is $1/t^2$. So the equivalent speed in the case of two-dimensional rotation turns out to be $(1/t^2)/t^2 = 1/t^4$. As could be seen, this is the fourth power expression of the corresponding time-space region speed $1/t$. (Note that in the time-space region time is *scalar* and there cannot be anything like two-dimensional time.)

Looking back, we can now easily see why the quantum theorists required *complex numbers* to deal with the so-called "electronic energy levels" of the atom adequately: they needed to cope up with the two-dimensional character of the equivalent speed pertaining to the one-dimensional rotation in the time region. It also suggests itself that we require to adopt *quaternions* to handle the so-called "nuclear energy levels" since the dimensionality of the equivalent speed pertaining to the two-dimensional rotation in the time region is four.

5.2 Potentials in the Time-space Region

At this stage of our study we have only two scalar motions (forces) to consider: the space-time progression and gravitation. In the outside region (the time-space region), the forces due to the space-time progression and gravitation are respectively given by

$$\left.\begin{aligned} F_{PO} &= K_{PO} \\ F_{GO} &= -\frac{K_{GO}}{r^2} \end{aligned}\right\} \tag{23}$$

where all the quantities concerned are in the natural units, the K's are positive constants and r the distance factor. Suffix G refers to gravitation, P to space-time progression and O to outside region. From the definition of potential, $F = -\partial V/\partial r$, we obtain the expressions for the corresponding potentials due to the space-time progression and gravitation, in the outside region respectively as

$$\left.\begin{aligned} V_{PO} &= -K_{PO} \cdot r \\ V_{GO} &= -\frac{K_{GO}}{r} \end{aligned}\right\} \tag{24}$$

The potential due to the space-time progression is repulsive while that due to gravitation is attractive as can be seen.

5.3 Potentials in the One-dimensional Zone of the time region

Potential energy being inverse speed, the expressions for the potentials in the one-dimensional zone of the time region would be the second power expressions of the corresponding ones in the time-space region (Section 5.1). Consequently the space-time progression and gravitational potentials in this zone could be written as

$$\left.\begin{aligned} V_{P1} &= K_{P1} \cdot r^2 \\ V_{G1} &= \frac{K_{G1}}{r^2} \end{aligned}\right\} \tag{25}$$

with suffix *1* referring to the one-dimensional zone. We can at once verify that gravitation is repulsive and the space-time progression attractive in this region. In addition there could be a constant term K_{I1}, representing the initial level of the time region potential. Thus the total time region potential in the one-dimensional zone turns out to be

$$V_{T1} = K_{P1} \cdot r^2 + \frac{K_{G1}}{r^2} \pm K_{I1} \tag{26}$$

The values of K_{G1} and K_{I1}, and possibly K_{P1}, are functions of the displacements of the atom in the three scalar dimensions.

It is instructive to see what the expressions for the corresponding *forces* would be: differentiating with respect to *r* and taking the negative sign, we have

$$\left.\begin{aligned} F_{P1} &= -2 \cdot K_{P1} \cdot r \\ F_{G1} &= \frac{2 \cdot K_{G1}}{r^3} \end{aligned}\right\} \tag{27}$$

Larson[10] however, while calculating the inter-atomic distances in solids, basing on the equilibrium of the time region forces, adopts

$$\left.\begin{aligned} F_{P1} &= -1 \\ F_{G1} &= \frac{K}{r^4} \end{aligned}\right\} \tag{28}$$

where *K* is a function of the several atomic rotations. These expressions can be seen to differ from Equations (27) above. But whether we take Equations (27) or Equations (28), the force equilibrium equation, $F_{P1} = F_{G1}$ can be seen to lead to the same fourth power dependence on the distance factor. Consequently, even if we find that Equations (27) are to adopted in preference to Equations (28), Larson's original inter-atomic distance calculations would remain unaltered.

The time region potential Equation (26) results in a potential well and therefore the solutions of Schrödinger's Equation (18) yield a set of discrete energy levels for the atomic system (see Section 3.4). It remains to be verified whether these truly correspond to the values inferred from the spectroscopic data.

5.4 Potentials in the Three-dimensional Zone of the time region

Turning now to the potentials in the three-dimensional zone, following our earlier analysis of the dimensional situation (Section 5.1), we adopt the fourth power expressions of the corresponding outside region (that is, the time-space region) quantities from Eqs. (24)

$$\left.\begin{aligned} V_{P3} &= K_{P3} \cdot r^4 \\ V_{G3} &= \frac{K_{G3}}{r^4} \end{aligned}\right\} \tag{29}$$

with suffix *3* denoting the three-dimensional zone.

We know that the space-time progression acts away from unit space. In the time-space region away

10 Larson, Dewey B., *Basic Properties of Matter*, *op. cit.*, p. 8.

from unit is also away from zero (the origin of the conventional spatial reference frame), whereas in the time region (that is, in less than unit space) away from unit is *toward* zero. This is the reason why the space-time progression is an outward motion in the outside region while it is inward in the time region. This is true in the one-dimensional zone of the time region as much as in the three-dimensional zone. But the "unit" of the three-dimensional zone does not coincide with the "unit" of the one-dimensional zone. Its boundary is determined by the apparent size of the atom in question. This is because the atom and the three-dimensional zone are *one and the same thing.* (We must avoid falling into the trap of imagining that first there is an atom, and that it "occupies" the pre-existing three-dimensional zone!) In Equation (7) of the article on Wave Mechanics[1] we have derived the following expression for the size of the atom,

$$r_A = 1.2 \times A^{1/3} \quad \text{femtometers}$$

where A is the atomic weight. Expressing this in the natural units as r_{An}, we now note that the reference point for reckoning distance in the case of V_{P3} is not the origin of the reference system but the point at r_{An}. Finally, since the potential due to progression has to be attractive a minus sign has to be introduced. Thus the expressions for the two potentials are

$$\left.\begin{aligned} V_{P3} &= -K_{P3} \cdot (r_{An} - r)^4 \\ V_{G3} &= \frac{K_{G3}}{t^4} \end{aligned}\right\} \tag{30}$$

Adding a constant term K_{I3} to take care of initial level of the potential energy, we have the total expression for the potential of the three-dimensional zone of the time region as

$$V_{T3} = -K_{P3} \cdot (r_{An} \cdot r)^4 + \frac{K_{G3}}{r^4} \pm K_{I3} \tag{31}$$

We note that this corresponds to what the conventional Quantum theorists would call the nuclear potential. Our study indicates that Equation (31) bears a remarkably close qualitative resemblance to the potentials arrived at through the scattering experiments. An unexpected feature of the experimental data analysis was the occurrence of a *repulsive core* of small radius. The Reciprocal System, on the other hand, actually predicts this repulsive core, namely, V_{G3}.

6 Conclusions

Let us summarize the highlights. Having resolved the riddle of the wave-particle duality in an earlier article[1] and understood the legitimacy of the wave picture in the Quantum theory, attempt has been made to examine the foundation of its mathematical formalism with the benefit of our knowledge of the Reciprocal System. This proved productive in two ways: firstly it clarified the situation in connection with the Quantum Mechanics, identifying some of its conceptual errors. Secondly it gave scope to expand our knowledge of the Reciprocal System in the form of new insights that would not have been possible otherwise.

(i) The Schrödinger equations were found to be valid general rules for the exploration of the wave functions in the various situations.

(ii) In the time-space region, speed can be vectorial (that is, "directional" in the context of the three-dimensional spatial reference frame), whereas inverse speed (like, energy) is scalar. In the time region, speed is found to be scalar, whereas inverse speed is directional—directional in the three-dimensional *temporal* reference frame. Variables of the latter type, therefore, could take on inherently negative values and be represented by complex numbers or quaternions as the case may be.

(iii) The penetration of the wave associated with particle into the regions of negative kinetic energy resulting from *potential energy barriers* is found to be a genuine time region phenomenon.

(iv) In a similar vein, it is found that the occurrence of a *well* type potential energy function in the time region leads to the limiting of possible values of total energy to a discrete set.

(v) Such an important empirical law as Pauli exclusion principle, which has no theoretical explanation in the context of the conventional theory, could easily be understood from the knowledge of the positive and negative reference points brought to light by the Reciprocal System.

(vi) Reasoning from the principles of the Reciprocal System the possible potential energy functions of the time region relevant to atomic systems are surmised. While they evince a close qualitative resemblance to the empirically found potentials, detailed further study needs to be carried out to see if they lead to the correct prediction of the properties pertaining to spectroscopy, radioactivity and the scattering experiments.

On the whole there seems to be a *prima facie* case in favor of adopting the Quantum Mechanics after purging it of its conceptual errors.

Appendix I: Euler's Relations

Often calculations are facilitated by adopting exponential functions with *imaginary* arguments in place of the sine or cosine functions, making use of Euler's relations

$$e^{ia} = \cos a + i \sin a$$
$$e^{-ia} = \cos a - i \sin a$$

which directly follow from the series expansions of these functions.

A number containing *imaginary* as well as *real* parts is called a *complex number.* Complex numbers may be represented graphically on a rectangular coordinate system, with the real part corresponding to the horizontal axis and the imaginary part to the vertical axis. Any complex number can then be represented by a vector extending from the origin and inclined at the angle a to the real axis. Thus $A \cdot e^{i\omega t}$ represents a (radial) vector of magnitude A rotating at the angular speed ω (t being time). It may be noted that *each* of the inverse relations,

$$\sin a = \frac{\left(e^{ia} - e^{-ia}\right)}{2i}$$
$$\cos a = \frac{\left(e^{ia} + e^{-ia}\right)}{2}$$

represents a *birotation*.

Appendix II: The General Equation of a Constant Speed Wave

Let a wave of arbitrary but unchanging shape be traveling in the X-direction of the stationary reference frame X-Y at a constant speed u. This wave appears stationary in a reference frame X_1-Y_1 which moves at the same speed u along the X-direction. We can then write

$$x_1 = x - u \cdot t \; ; \; y_1 = y \tag{32}$$

If the wave shape in the co-moving frame is given by $y_1 = f(x_1)$, we have from Equation (1)

$$y = f\left(x - u \cdot t\right) \tag{33}$$

By the chain rule for derivatives we have

$$\frac{\partial y}{\partial x} = \frac{dy}{dx_1} \frac{\partial x_1}{\partial x} = \frac{dy}{dx_1} \cdot 1,$$

$$\frac{\partial y}{\partial x} = \frac{dy}{dx_1} \frac{\partial x_1}{\partial x} = \frac{dy}{dx_1} \cdot (-u).$$

Therefore the relation between the two derivatives is

$$\frac{\partial y}{\partial x} = -\frac{1}{u} \cdot \frac{\partial y}{\partial t} \tag{34}$$

Similarly for a wave traveling in the -X direction we obtain

$$\frac{\partial y}{\partial x} = +\frac{1}{u} \cdot \frac{\partial y}{\partial t} \tag{35}$$

Now a repeated application of the above procedure yields

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{u^2} \cdot \frac{\partial^2 y}{\partial t^2} \tag{36}$$

which is the governing equation of the wave function; and it is the same for waves traveling in either direction of the X-axis.