New Light on the Gravitational Deflection of Radiation Path

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I have discussed in an earlier paper,[1] the effect of gravitation on the bending of the locus of a photon. Even though the role played by coordinate time associated with gravitational motion in deflecting the path of light was correctly depicted therein, I believe that the mathematical implications were not correctly brought out. Especially, in the case of Equation (6) there, though its existence was correctly recognized, its physical significance was misconstrued. The present paper, therefore, attempts to overcome these shortcomings and derives the mathematical expression for the angle through which the path of light beam deflects in the vicinity of a mass.

According to the *Reciprocal System* an independent motion of speed v has associated with it an increase in coordinate time amounting to $(v/c)^2$ sec/sec (c being the speed of light). In the case of a one-dimensional motion, like that of a planet orbiting the sun or of a photon grazing the sun's limb, I have pointed out[2] that the circumferential spatial effect arising out of coordinate time amounts to $3(v/c)^2$ sec/sec. It was further explained that, in the case of photon, this spatial effect manifests in the radially inward direction since no further circumferential effect is possible as the photon is already moving at unit speed in the latter direction.[3]

The gravitational speed v at any radial distance r from a mass M is shown[4] to be

$$v^2 = \frac{GM}{r}$$

where G is the gravitational constant. Thus, we have the rate of coordinate time increase at a radial distance r outside a mass M as

$$\frac{dt_c}{dt} = 3\left(\frac{v}{c}\right)^2 = \frac{3GM}{rc^2} \tag{1}$$

where t_c represents the coordinate time and t the clock time.

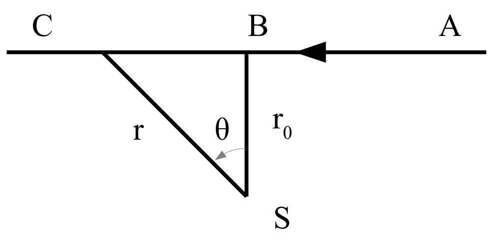

Figure 1

As shown in Figure 1 let the straight line ABC represent the locus of a photon passing the sun situated

1 K. V. K. Nehru, "Gravitational Deflection of Light Beam in the Reciprocal System," *Reciprocity* XI(1), Spring 1981, p. 28.

2 K. V. K. Nehru, "Precession of the Planetary Perihelia Due to the Co-ordinate Time," *Reciprocity* XIV(1), Autumn, 1985, p. 11.

3 K. V. K. Nehru, "Gravitational Deflection…," *op. cit.*, p. 29.

4 *Ibid.*, Equation (1), p. 28.

at S. With SB perpendicular to AC, B is the point of closest approach to the sun. Let SB = r_0 . The equation of the line ABC in polar coordinates, with the center at S, is given by

$$r_0 = r\cos(\theta) \tag{2}$$

where *r* is the radial distance at any angle θ measured counterclockwise from SB. Substituting *r* from Equation (2) into Equation (1), we have

$$\frac{dt_c}{dt} = \left(\frac{3GM}{r_0 c^2}\right)\cos(\theta) \tag{3}$$

Now we note that the gravitational effect of any mass aggregate, according to the *Reciprocal System*, does not extend up to infinity, but becomes zero at a limiting distance, which Larson calls the "outer gravitational limit," d_1. As such, we need to compute the coordinate time increase in the case of the transiting photon, starting from the outer gravitational limit on one side (toward A, in Figure 1), up to the outer gravitational limit on the other side (toward C, in Figure 1). Larson worked out the value of the outer gravitational limit for the sun to be nearly 13350 lightyears.[5] As this will be very large compared to r_0, we find that the limits on the two sides are given by $\theta_1 = -\pi/2$ and $\theta_2 = +\pi/2$.

Hence, using Equation (3), the average rate of coordinate time increase during this transit from θ_1 to θ_2 is given by

$$\begin{aligned} \frac{dt_c}{dt}\Big|_{av} &= \frac{\left(\int_{\theta_1}^{\theta_2} \frac{dt_c}{dt}\, d\theta\right)}{(\theta_2 - \theta_1)} \\ &= \left(\frac{3GM}{r_0 c^2}\right)\int_{-\pi/2}^{\pi/2} \cos(\theta)\,\frac{d\theta}{\pi} \\ &= \frac{6GM}{\pi r_0 c^2} \quad \textit{sec/sec} \end{aligned} \tag{4}$$

Since the total distance traveled is $2d_1$, to total transit time is

$$t = \frac{2d_1}{c} \quad \textit{sec} \tag{5}$$

Therefore, the total coordinate time gained during this clock time *t* is

$$\begin{aligned} t_{c,tot} &= \frac{t \times dt_c}{dt}\Big|_{av} = \left(\frac{6GM}{\pi r_0 c^2}\right) \times \left(\frac{2d_1}{c}\right) \\ &= \frac{12GMd_1}{\pi r_0 c^3} \quad \textit{sec} \end{aligned} \tag{6}$$

5 Larson, *Universe of Motion*, North Pacific Publishers, OR., 1984, p. 201.

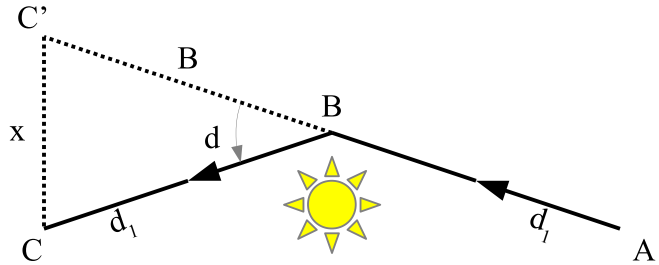

Figure 2: Sun

In Figure 2 the directions of approach and departure of the light beam are shown as ABC' and BC respectively. CC' represents the spatial shift in the radial direction arising out of the coordinate time component and is given by

$$x = c \times t_{c,tot} = \frac{12 G M d_1}{\pi r_0 c^2} \quad cm \tag{7}$$

Finally, from Figure 2 we can sec that the angular deflection, according to the *Reciprocal System*, is given by

$$d_{RS} = \frac{x}{d_1} = \frac{12 G M}{\pi r_0 c^2} \quad radians \tag{8}$$

The corresponding expression from the General Relativity is

$$d_{GR} = \frac{4 G M}{r_0 c^2} \tag{9}$$

The discrepancy between the two formulae can be seen to be

$$\frac{d_{RS}}{d_{GR}} = \frac{3}{\pi} = 0.955 \tag{10}$$

The value calculated from the *Reciprocal System* formula, for the sun, is 1.67 arcsec, whereas the General Relativity value is 1.75 arcsec. The reported values vary from 1.5 to 1.8 arcsec.