The Gravitational Limit and Hubble's Law

Prof. K.V.K. Nehru, Ph.D.

1 The Gravitational Limit

Gravitation and Space-Time Progression (STP) are the two oppositely directed scalar motions that decide the outcome of all physical phenomena in the universe of motion. There are two hitherto unknown features of gravitation that the Reciprocal System has brought to light. The first one, which is relevant to atomic-scale phenomena, is that in the context of the familiar three-dimensional stationary frame of reference, the direction of gravitation reverses at the unit space limit: it manifests as a repulsive force in the time region—the region inside unit space. This phenomenon forms the basis of cohesion in solids.

The second feature is relevant to large-scale phenomena and is concerned with gravitation of mass aggregates. Even though the net total gravitational motion of a material aggregate is constant, the *effective* magnitude of its force aspect is attenuated by the inverse-square law in the context of the three-dimensional stationary frame of reference. On the other hand, the magnitude of the force aspect of the STP is independent of distances since the progression originates at every location of the reference frame. Consequently, Larson points out, "…the gravitational limit of a mass is the distance at which the inward gravitational motion of another mass toward the mass under consideration is equal to its outward motion due to the progression of the natural reference system relative to our stationary system of reference… ."[1] Thus, the net motion inside the gravitational limit is inward, while outside it is outward.

Let us consider a spherical aggregate of mass *M*. The force due to its gravitational motion acting on a unit mass situated at a distance *x* (outside of it) is given by

$$a_g = \frac{-GM}{x^2} \quad \textit{dynes/gm or cm/s}^2 \tag{1}$$

where *G* is the "universal" constant of gravitation. The minus sign implies that the force is directed inward (that is, tending to decrease intervening distance).

In a similar manner, we can write that the outward force on the unit mass due to progression as

$$a_p = P \quad \textit{dynes/gm or cm/s}^2 \tag{2}$$

where *P* may be referred to as the universal constant of progression. Thus, the net force per unit mass (that is, acceleration) at a distance *x* from a mass *M* is given by

$$a_n = a_p + a_g = P - \frac{GM}{x^2} \tag{3}$$

Larson[1] evaluates the gravitational limit adopting the magnitudes of the quantities concerned in natural units. In natural units, the so-called "constants of nature" do not occur, these being the result of an arbitrary choice of conventional units. Thus, the gravitational force, in natural units, due to a mass *M* at

1 Larson, Dewey B., *Universe of Motion* (North Pacific Publishers, OR, 1984), p. 195.

a distance x (both in natural units) is simply M/x^2. And since the force due to progression, again in natural units, is unity, the gravitational limit d_0 is evaluated from the relation

$$\frac{M}{d_0^2}=1 \tag{4}$$

However, there is another factor to be considered. Gravitation is the translatory aspect of the scalar rotation that constitutes units of matter (atoms). The rotation exists within unit space (the time region), whereas the linear translatory effect manifests in three-dimensional space (the time-space region). We have shown elsewhere[2] that the atomic rotation is distributed over $156.4\bar{4}$ degrees of freedom in the time region. In addition, linear translatory motion in the three-dimensional spatial region is distributed over 8 degrees of freedom. As such, the number of rotational units (mass units) in the time region that are in equilibrium with a unit of linear translation in the time-space region is 156.44 × 8. But it will be recalled that gravitation is a three-dimensional motion: in fact, a three-dimensional inverse speed.[3] In terms of space-time units, its natural dimensions are t^3/s^3. Hence, the total number of possibilities over which the gravitational effect of a unit of mass is distributed is $(156.44 \times 8)^3$. Considering this, Equation (4) above has to be rewritten as

$$\frac{M}{(156.44\times 8)^3}=d_0^2$$

in natural units.

Adopting the values of 1.65979×10^{-24} g, and 4.558816×10^{-6} cm for the natural units of mass and length respectively from Larson,[4] we have the gravitational limit of a mass aggregate M as

$$d_0=3.77\left(\frac{M}{M_0}\right)^{1/2} \textit{light-years} \tag{5}$$

where M_0 is the mass of the sun.

Using Equation (3) and setting $a_n = 0$ at the gravitational limit, we can now evaluate the force due to STP as applicable to aggregate phenomena (and in the context of a three-dimensional stationary reference frame) as

$$P=\frac{GM}{d_0^2} \tag{6}$$

$$=1.044\times10^{-11}\ \textit{dyne/gm} \tag{6-a}$$

2 Speeds in the region inside the gravitational limit

We can obtain the expressions for the speeds due to gravitation and progression respectively from Equations (1) & (2). If v_g is the gravitational speed and t the time, from Equation (1),

2 K.V.K. Nehru, "The Inter-regional Ratio," *Reciprocity* XVI (2-3), Winter 1985-1986, p. 5.
3 Larson, Dewey B., *Nothing But Motion*, (North Pacific Publishers, OR, 1979), p. 147-148.
4 *Ibid.*, p. 160.

$$a_g = \frac{dv_g}{dt} = \left(\frac{dv_g}{dx}\right) \times v_g = \frac{-GM}{x^2}$$

On integrating, we get

$$v_g^2 = \frac{2GM}{x} + b_1 \tag{7}$$

where b_1 is the constant of integration. Similarly, taking v_p as the speed due to progression, from Equation (2) we have

$$a_p = \left(\frac{dv_p}{dx}\right) \times v_p = P$$

On integrating, we have

$$v_p^2 = 2Px + b_2 \tag{8}$$

with b_2 as the constant of integration. Taking, at the gravitational limit ($x = d_0$), the net speed v_n ($= v_p - v_g$) to be zero, we have $v_p = v_g$. Hence we obtain from Equations (7) & (8)

$$b_1 - b_2 = 2\left(P d_0 - \frac{GM}{d_0}\right) = 2 d_0 \left(P - \frac{GM}{d_0^2}\right)$$

Substituting from Equation (6) we finally have $b_1 = b_2$. Since b_1 and b_2 pertain to two motions of altogether different origins, one possibility that immediately suggests itself is that $b_1 = b_2 = 0$. In fact, empirical evidence (on the velocity of escape) validates this possibility. We can now write down the expression for the gravitational speed from Equation (7). We will find it convenient to have the equations in non-dimensional form. Therefore, we write

$$v_g = \left(2\frac{GM}{x}\right)^{1/2} = \left(2\frac{GM}{d_0}\right)^{1/2} \times \left(\frac{d_0}{x}\right)^{1/2} = \frac{v_0}{y^{1/2}} \tag{9}$$

where $y = x/d_0$, the distance in non-dimensional form, and

$$v_0 = \left(2\frac{GM}{d_0}\right)^{1/2} \tag{9-a}$$

which we shall henceforth refer to as the "zero-point speed" of the mass aggregate. The zero-point speed v_0, may be viewed as the gravitational speed that is in equilibrium with the STP at the equilibrium distance d_0. It has the significance of being the natural unit of speed germane to a mass aggregate. It serves the same function in the case of mass aggregates as is served by *c*, the speed of light in the case of individual mass units.

Using Equations (8) & (6), we can write the speed due to progression as

$$v_p = v_0 \times y^{1/2} \tag{10}$$

Finally, the net speed in the region inside the gravitational limit ($y <= 1$), but outside the mass M is given by

$$v_n = v_p - v_g = v_0\left(y^{1/2} - \frac{1}{y^{1/2}}\right) \tag{11}$$

It may be noted that for distances considerably smaller than the gravitational limit, this equation reduces to

$$v_n = \frac{-v_0}{y^{1/2}} \tag{11-a}$$

(the minus sign implying that the speed is inward.)

3 Speeds in the region outside the gravitational limit

An examination of Equation (11) shows that while the net speed is negative (radially inward) within the gravitational limit, it is positive or outward in the region beyond it. The crucial point that must be recognized at this juncture is that "…the three-dimensional region of space extends only to the gravitational limit… beyond this limit… the gravitational effect of the aggregate… is in equivalent space rather than in actual space."[5] Larson further points out that all quantities in equivalent space are two dimensional in terms of actual space.[6] Therefore, in order to obtain the speeds pertaining to the region beyond the gravitational limit, we have to take the respective second power expressions.

First, we convert the gravitational speed v_g into natural units by dividing the zero-point speed v_0 of the aggregate, and then square it. Thus, the expression for the gravitational speed in the outer region is $(v_g/v_0)^2$. However, as this is two-dimensional, the effective speed in the dimension of the time-space region is half of this quantity. Therefore, the gravitational speed for *x* greater than d_0 is given by

$$\begin{aligned} v_{g^0} &= \frac{1}{2}\left(\frac{v_g}{v_0}\right)^2 \quad \textit{in natural units} \\ &= \frac{1}{2}\left(\frac{v_g}{v_0}\right)^2 \times v_0 \quad \textit{in cm/s}^2 \end{aligned}$$

Substituting for v_g/v_0 from equation (9)

$$v_{g^0} = \frac{1}{2}\frac{v_0}{y} \tag{12}$$

Following similar procedure, we obtain from Equation (10) the speed due to STP effective in the outer region as

$$v_{p^0} = \frac{1}{2} v_o \times y \tag{13}$$

Finally, the net speed in the outer region is

5 Larson, Dewey B., *Universe of Motion*, *op. cit.*, p. 197.
6 *Ibid.*, p. 210.

$$v_{n^0} = \frac{1}{2} v_0 \left(y - \frac{1}{y} \right) \tag{14}$$

4 Hubble's Law

It can readily be seen that for distances large compared to the gravitational limit, equation (14) reduces to

$$v_{n^0} = \frac{1}{2} v_0 \times y \tag{14-a}$$

where v_{n0} is an outward speed. Substituting for v_0 and y in terms of the original variables

$$v_{n^0} = \frac{1}{2} \left(\frac{2GM}{d_0} \right)^{1/2} \left(\frac{x}{d_0} \right) = \left(\frac{GM}{2d_0^3} \right)^{1/2} \times x \tag{15}$$

This is identical to Hubble's law of the recession of the distant galaxies:

$$v_r = H \times x$$

Hubble's constant turns out to be

$$H = \left(\frac{GM}{2d_0^3} \right)^{1/2} \tag{16}$$

Substituting for d_0 from Equation (5), we note that Hubble's constant is inversely proportional to the fourth root of the galactic mass. Thus, if *M* is in solar mass units,

$$H = \frac{37302.19}{M^{1/4}} \quad km\ s^{-1}\ Mpc^{-1} \tag{16-a}$$

The value of Hubble's constant could be calculated from the above equation, if we know the mass of our galaxy accurately. As it turns out, it is the Hubble constant that we know with less uncertainty than the mass of the galaxy. We will, therefore, calculate the mass of the galaxy from the above equation. Adopting the value H = 55 km s^{-1} Mpc^{-1}, we have the following results for our galaxy

$$M_G = 2.116 \times 10^{11} \quad solar\ mass\ units$$

and

$$d_{0G} = 0.532 \quad Mpc$$
$$v_{0G} = 58.5 \quad km\ s^{-1}$$

In passing, it may be remarked that Equation (14-a), which leads to the strictly linear form of Hubble's law, is not applicable to shorter distances comparable to d_0. For these distances, Equation (14) must be used. The discrepancy between the results of the two equations becomes significant for distances less than about 10 times d_0 (that is, about 5 Mpc in the case of our galaxy). However, whether the variation of the recession speed within this distance range follows the strictly linear law is not observationally verifiable. This is because the speeds of the peculiar motion of these nearby galaxies are commensurate with their recession speeds and since the former are random in nature, no conclusion is possible regarding the manner of variation of the recession speed of these nearby galaxies with the distance.

5 Summary

1. The gravitational limit of an aggregate of mass M is given by

   $$d_0 = 3.77\left(\frac{M}{M_0}\right)^{1/2} \text{ light years}$$

   where M_0 is the mass of the sun.

2. The value of the universal constant of progression for aggregate phenomena is

   $$P = 1.044\times10^{-11} \text{ cm/s}^2$$

3. The natural unit of speed for a mass aggregate, called the zero-point speed, is given by

   $$v_0 = \left(\frac{2GM}{d_0}\right)^{1/2} \text{ cm/s}$$

   where G is the universal constant of gravitation.

4. The net speed due to gravitation and progression outside of a mass aggregate of mass M, at a distance x is given by

   $$v_n = v_0\left(y^{1/2} - \frac{1}{y^{1/2}}\right) \text{ for } y \le 1.0$$

   $$v_{n^0} = \frac{1}{2} v_0\left(y - \frac{1}{y}\right) \text{ for } y \ge 1.0$$

   where $y = x/d_0$.

5. The recession speed of distant galaxies ($x > 10\ d_0$) is given by

   $$v_r = H \times x$$

   H the Hubble's constant = $\left(\frac{GM}{2d_0^3}\right)^{1/2} = \frac{37302.19}{\left(\frac{M}{M_0}\right)^{1/4}}$ km s^{-1} Mpc^{-1}

   M being the mass of our galaxy.

6. The results calculated for our galaxy on the basis of H = 55 km.s^{-1}.Mpc^{-1} are

   mass = 2.116×10^{11} solar units

   gravitational limit = 0.532 Mpc, and

   zero-point speed = 58.5 km.s^{-1}