Time of Planetary Perihelion Motion

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1 Introduction

Perihelion is the name for a discrete location a planet occupies momentarily once during every cycle of its periodic motion about the sun; the planet when at this location is nearest the sun. The location does not remain the same from one cycle to the next. Since successive cycles occur at different times, the location changes with the progress of time. This displacement of place with time constitutes perihelion motion of a spatial location at definite, measurable time and motion rates.

The object of this paper is twofold:

A. to show that the well-known excess precession rate of Mercury's perihelion is a function not of gravitation, as supposed by the general relativity theory of Einstein[1], but rather of the three-dimensional and discrete character of time (and space) and

B. to indicate that the time involved in perihelion motion is not merely clock time[2], as is generally assumed.

2 Space, Time, Mass and Motion

It is unwise to forget that time and space are terms which were drafted into natural philosophy and science to measure motion; they were not introduced initially because of their presumed measurability with clocks and rigid rods.

In practice the accepted measure of motion rate (velocity, speed, including angular velocity and frequency), has always been a reciprocal relation between space and time. If spatial displacement be denoted by s and time progression by t, then

$$\text{Motion Rate} = \frac{s}{t} \tag{1}$$

The concept of mass, without which even ideal rigid rods and clocks cannot be properly defined, is not required to define motion rate. Mass together with motion rate is essential to define linear and angular momentum, a concept required to measure material motion (the motion of matter).

It is occasionally said that only matter moves. It is said also that the existence of both time and space necessarily depends on the prior existence of matter and mind. Such statements are quite arbitrary assumptions and not necessarily truths, still less are they necessary truths.

The elementary meaning of time and space from the evidence at hand is that they are the two essential

1 Einstein, A. "Der Grundlage der allgemeinen Relativitats theorie," Annalen der Physik 49, 1916, translated in *The Principle of Relativity*, Dover, (Methuen Co. 1923).

2 Larson, D. *New Light on Space and Time*, North Pacific Publishers, Portland, Oregon, 1965.

aspects of every kind of motion. Every kind of motion includes all motions of the atoms of matter and also other kinds of motion, such as planetary perihelion motions (which are motions of locations). Time and space have no immediate meaning from this viewpoint apart from their use to represent motion rate, as expressed in Equation (1). The latter assumption is the least arbitrary that can be made. This assumption is in complete, uncontradicted accord with long accepted elementary usage of the two terms in the practice and theory of physics and engineering. The same usage prevails probably in all other human languages as well as in English.

Accurate conception of time and space is so fundamental to physics that, if any incorrect arbitrary assumptions are made about them along the theoretical way, these misconceptions will inevitably lead the science up a tree. The surfeit of mathematics that fills the current world's physics journals cannot rescue theory predicated upon such misconceptions from this proverbial tree. Reams of calculations cannot indefinitely patch over erroneous underlying principles. The historical record shows an interesting parallel in the case of the scientific theory which perhaps has held more sway over the minds of at least Western humankind than any other—the Aristotelian-Ptolemaic theory of the physical universe. Neither circles nor epicycles, equants and eccentrics, etc. could save this scientific theory from its basic conceptual error of supposing that the actual planet earth is immovably located at the supposed center of the actual physical universe.

3 Magnitude of Mercury's Perihelion Precession

The planet Mercury's perihelion advance exceeds by almost twenty miles per revolution the amount predicted by Newton's theory of gravitational motion. Years ago astronomers calculated on the basis of Newtonian theory that Mercury's perihelion would rotate at a rate of 532 seconds of arc length every 100 years in an inertial frame (5,556.6 seconds of arc length per century in a non-inertial frame fixed on an equinox). Meanwhile, the astronomers' carefully measured observations have disclosed that it actually precesses at the larger rate of 575 seconds of arc length per century, an excess of 43 seconds of arc length above the Newtonian theoretical value (for a total of 5,599.6 seconds of arc/century in the non-inertial frame). In perhaps more understandable and equivalent language this amounts to saying that for the perihelion and major axis of Mercury's elliptical orbit to complete one revolution about the sun takes about three million years. The planet itself takes about 88 earth days to go once around the sun.

3.1 Why Reexamine Planetary Perihelion Motion Theory?

Two good reasons for reopening a discussion of planetary perihelion motion are to learn:

1. precisely where the fundamental assumptions of Newton's theory of space, time and gravitational motion are in error and
2. whether Einstein's general relativity theory of gravitation is right.

Newton treated planetary perihelion motion as simply a gravitational effect. According to his conception of gravitational motion, the perihelion and elliptical orbit of a single planet revolving about a spherical sun would be permanently fixed in space. The occurrence of a second planet or additional planets would cause the perihelion of the first planet to advance. In the case of Mercury, therefore, its computed perihelion advance of 532 seconds of arc/century was attributable to the combined effects of its neighboring planets in the solar system.

The problem of how this Newtonian-inspired computation fell 43 seconds of arc/century short of the observed mark is rooted in Newton's assumptions and misconceptions about time, space and motion rather than in his law of gravitation. Newton[3] seems to have conjectured that space, time and motion are each separable from and prior to matter.

He seems also to have supposed that time and space are distinguishable from one another. He appears to have assumed further that space and time are two continua, separable from one another, and unequally related to motion. He finally appears to have supposed that time is a one-dimensional, uniform and continuous flow and that space is an absolutely immovable three-dimensional continuum.

Another purpose in reexamining planetary perihelion motion is to alert physicists and astronomers to reopen the question whether a satisfactory explanation of the motional anomaly of Mercury's perihelion has been found, particularly in Einstein's theory of general relativity.

It is widely held that the Einstein theory satisfactorily explains the excess 43 seconds of arc/century motion of Mercury's perihelion.

This theory assumes that the additional motion of Mercury's perihelion is essentially another gravitational effect. The assumption, of course, has not been proved and is questionable. It is true that the general relativity theory does produce good agreement between its calculation and the observation of the excess perihelion precession rate. However, although mathematical agreement with observed measures is a necessary condition to prove a physical theory true, it is not a sufficient condition. The conceptual validity of a physical theory never can be mathematically demonstrated. Since this situation is not always recognized, a comment by Bridgman[4] is cited to reiterate it:

> How is it then, if the arguments on which the general theory rests are open to these criticisms, that the theory has given correct results, and in particular led to correct predictions with regard to the advance of the perihelion of Mercury? As I have already emphasized, what the theory says about itself is not pertinent, and an incorrect argument may lead to the same demands as other less objectionable arguments might lead to.
>
> This, it seems to me, is what has happened here.

Be that as it may, the reputation of Einstein's general relativity theory hinges particularly on its presumed adequacy to explain Mercury's perihelion motion. Claims made by the theory to explain additional phenomena would be affected, if the theory were found insufficient to explain the excess motion of Mercury's perihelion.

1. The relativistic explanation of Mercury's excessive perihelion precession rate will be reviewed.
2. Then a conceptually different but mathematically equivalent explanation will be presented.
3. Finally, a test will be proposed by which we can learn which of the two theories more truly represents the actual character of perihelion motion, if either of them truly does.

General Relativity Account of Perihelion Motion

The additional 43 seconds of arc length per century in the observed rate of 575 seconds of arc/century for Mercury's perihelion was accounted for by the general relativity theory as follows:

The mass of a planet was said to increase as its velocity increases. Since the planet's velocity is

3 Newton, I. *Mathematical Principles of Natural Philosophy*, translated by Andrew Motte (1729), revised by Florian Cajori (Berkeley, UC Press, 1934).

4 Bridgman, P. W., *The Nature of Physical Theory*, Science Editions, John Wiley & Sons, Inc., NY. 1964.

maximum at perihelion, the mass of the planet therefore was said also to become maximum when the planet is at perihelion. Consequently, Einstein inferred that there would be an additional perihelion precession increment from the increased gravitational pull of the sun on the increased planetary mass at perihelion. He concluded that this extra pull by the sun would have the effect of causing the space-time location of the perihelion to advance, even if there was no second or third planet.

Einstein rejected Newton's arbitrary assumption that the existences of space and time are separable. In developing a formula for calculating the self-procession rate of a planetary perihelion he introduced several new arbitrary assumptions among which are: a) motion and matter are inseparable; b) space and time form a single continuum in which they are distinguishable only dimensionally; c) time can be spatialized and geometrized and d) the space-time continuum structure is a function of matter.

Einstein's general relativity theory specifically relates planetary perihelion precession rate to the eccentricity of a planet's elliptical orbit. The rate is said to be the larger, the more eccentric the orbit, if the mean orbital velocity of the planet is unchanged. This result was reached by Einstein[1] in the last of seventy-five equations contained in an early paper on his general relativity theory. Einstein expressed the result thus:

> … we find a deviation of the following kind from the Kepler-Newton laws of planetary motion. The orbital ellipse of a planet undergoes a slow rotation, in the direction of motion, of amount
>
> $$R=\frac{24\pi^3 a^2}{T^2 c^2\left(1-e^2\right)} \tag{75}$$
>
> per revolution. In this formula a denotes the major semi-axis, c the velocity of light in the usual measurement, e the eccentricity, T the time of revolution in seconds.
>
> Calculation gives for the planet Mercury a rotation of the orbit of 43" per century, corresponding exactly to astronomical observation (Leverrier); for the astronomers have discovered in the motion of the perihelion of this planet, after allowing for disturbances by other planets, an inexplicable remainder of this magnitude.

The equation numbered 75 implies that the excess precession rate increases with increasing orbital eccentricity. The equation says further that the rate increases without bound as the eccentricity approaches unity. The approach of the eccentricity to unity implies that an elliptical orbit approaches maximum possible eccentricity. An eccentricity equal to unity corresponds to a parabola.

It will be convenient to transform Einstein's equation 75, written in units of radians per cycle, into the following equivalent expression in units of fraction of a cycle per cycle:

$$R=\frac{12\pi^2 a^2}{T^2 c^2\left(1-e^2\right)}\ \frac{\textit{fraction of cycle}}{\textit{cycle}} \tag{2}$$

Equation (2) now can be simplified further by writing that

$$\frac{4\pi^2 a^2}{T^2}=V^2 \tag{3}$$

where V = the mean orbital velocity of the planet. Then

$$R = \frac{3}{c^2}\frac{4\pi^2 a^2}{T^2}\frac{1}{1-e^2} = \frac{3V^2}{c^2(1-e^2)} \quad \frac{\textit{fraction of cycle}}{\textit{cycle}} \tag{4}$$

From Equation (4) or its equivalent, his Equation 75, Einstein, as quoted above, calculated that the sun would produce an additional precession of Mercury's perihelion, amounting to 43 seconds of arc length per century.

If Equation (4) holds for Mercury, there seems no reason why it should not be applied to the other planets. Since Einstein's general relativity theory was published in 1916, measurements of the excess perihelion motion have been made on Venus and Earth as well as Mercury. Table I shows some of the best measures of the excess precession rates for the perihelia of these planets that are now available.

Table I: Perihelion Excess Precession Rate Measurements

Planet	**R (" of arc length/century) Clemence, 1947**[5]	**R (" of arc length/century) Duncombe, 1956**[6]
Mercury	42.56 ± 0.94	43.11 ± 0.45
Venus		8.4 ± 4.8
Earth	4.6 ± 2.7	5.0 ± 1.2

Temporal Account of Perihelion Motion

A theory of the excess motion of Mercury's perihelion, conceptually different from the relativistic account, is the temporal theory. This theory is a corollary of the reciprocal space-time concept of motion, proposed by Larson[2,7]. The temporal theory agrees with the Newtonian account, which attributes 532" of arc/century of Mercury's perihelion total motion rate of 575" of arc/century to the gravitational motions of the other planets. The temporal theory, however, takes issue with the Einsteinian account, which attributes the excess motion rate of 43" of arc/century to an additional gravitational effect said to result from a maximum increase of Mercury's mass at perihelion that in turn is said to follow from Mercury's observed maximum velocity at this location. Specifically, the temporal account rejects the implication of the general relativity theory that Mercury's perihelion motion rate explicitly depends on the eccentricity of the planetary orbit.

The temporal account states that Mercury's perihelion moves further than Newton's theory predicted, because the spatial location has more time to move than was admitted by the theories both of Newton and Einstein. According to the latter theories, the only time to be reckoned with is a one-dimensional continuous (infinitely divisible) time that can be measured with a clock. This clock time is the only component of time recognized by either author. Newton[3] seems correctly to have recognized that clock time rate is invariably uniform (time "flows equably"). Einstein, on the contrary, seems to have conceived that clock time rate must depend on the motion rate of the matter composing the clock. He said that clock time was not uniform, but rather clock time slowed, the faster the clock moved with respect to another clock. Subsequent to the Michelson-Morley experiment, the concept that clock time is total time has become questionable and untenable. The temporal theory asserts that besides clock

5 a. Clemence, *G.M. Rev. of Mod. Phys.* 19, 361, 1947; b. Clemence, *G.M. Proc. of Am. Philo. Soc.* 93, 532, 1949.
6 Duncombe, R.J. *Astron*. J. 61, 174, May, 1956.
7 Larson, D. *Beyond Newton*, North Pacific Publishers, Portland, Oregon, 1964.

time there is another component of total time, a coordinate time component. The temporal theory states that the coordinate time component of Mercury's perihelion motion can and does explain its excess motion rate of 43" of arc every 100 years. A brief exposition of the meaning of coordinate time and some related premises of the temporal theory is found in Appendix I.

The temporal theory differs most conspicuously from the relativity theory of excess perihelion motion in finding that the excess precession rate does NOT explicitly depend on a planet's elliptical orbital eccentricity.

Specifically, the equation of the temporal theory for computing excess perihelion motion is formulated as follows:

$$R = \frac{\pi V^2}{T c^2} \quad \frac{fraction\,of\ cycle}{cycle} \tag{5}$$

where R, V, T and c have the same significance as in Equations (2), (3) and (4).

Equation (5) does not explicitly contain an orbital eccentricity term e as do Equations 75, (2) and (4) of the general relativity theory. The significance of this difference is that Equation (5) of the temporal theory predicts smaller excess perihelion precession rates than the general relativity theory does for larger elliptical eccentricities that are found in the planets of the solar system. Here eccentricities less than but commensurable with unity are under consideration. The maximum eccentricity found for a planetary orbit in our solar system approximately equals 0.25.

There is a significant respect in which Equation (5) of the temporal theory and the equations of the general relativity show the same dependence of R implicitly on orbital eccentricity e. This situation arises from the dependence of R in both theories on the same function of orbital velocity V, namely V^2. As indicated in equation (3), V is a function of the elliptical semi-major axis a. Furthermore, orbital eccentricity e is a function of a, as shown in Appendix 2. Thus, perihelion excess precession rate R has an implicit dependence, the same in the two theories, on orbital eccentricity e. Comment on the significance of this implicit dependence of R on e is found in Appendix 2.

In Table II is given all the information about the period and mean orbital velocity for each planet needed to compute the excess precession rate of its perihelion with the aid of equation (5).

Table II: Comparison of Perihelia Excess Precession Rates
Computed by New Temporal Method
with Some Measured Rates and
with Rates Computed by General Relativity

Planet	V, Mean Orbital Velocity miles/second	T, Round Trip Period years
Mercury	29.7	0.2408
Venus	21.7	0.6152
Earth	18.5	1.0000
Mars	15.0	1.8808
Jupiter	8.1	11.862
Saturn	6.0	29.458
Uranus	4.2	84.015
Neptune	3.4	164.788
Pluto	3.	247.697

Results

Results of calculating perihelion motion rates according to the temporal equation, Equation (5), are shown in Table III together with available well-measured values of some of these rates. Values of the same rates calculated from the general relativity equation, Equation (4), also are listed in Table III.

Table III: Comparison of Perihelia Excess Precession Rates
Computed by New Temporal Method With Some Measured Rates
and With Rates Computed by General Relativity Method

Planet	R (Calculated) Temporal Method	R (Measured) Duncombe[6]	R (Calculated) Relativity Method
Mercury	43.15 arc sec/century	43.11 ± 0.45	43.03
Venus	8.97	8.4 ± 4.8	8.64
Earth	4.04	5.0 ±1.2	3.84
Mars	1.41	–	1.35
Jupiter	0.065	–	0.06
Saturn	0.014	–	0.014
Uranus	0.002	–	0.002
Neptune	0.001	–	0.001
Pluto	0.000	–	0.000

As Table III shows, the calculated values obtained with Equation (4), the general relativity equation, agree within the estimated experimental errors with available measured rates for the cases of Mercury, Venus and Earth. Likewise, the calculated values obtained from Equation (5), the temporal equation, evidently agree within the estimated experimental errors with available measured rates for the cases of Mercury, Venus and Earth. Measured rates have not been reported for any other planet at present. The results means that, although the temporal theory and the general relativity theory are utterly different in their conceptions of time, space and motion, they nevertheless are mathematically equivalent for computing accurately within the limits of reported experimental error the measured perihelion excess precession rate of every planet within our solar system.

Proposed Test For Estimating Truth Capacity of the Two Theories

The temporal Equation (5) and the general relativity Equation (4) are mathematically equivalent only to the degree that the following approximate equality remains valid:

$$\frac{\pi}{T} \approx \frac{3}{1-e^2} \tag{6}$$

From Equation (6) it can be inferred that the temporal theory could be alleged to be indistinguishable conceptually from the general relativity theory only if it were established in every case that planetary period, T, is the SAME constant function of orbital eccentricity e:

$$T = f(e) = \frac{\pi}{3(1-e^2)} = 1\,cycle \tag{7}$$

For T is always unity in Equation (6) as well as in Equations (4) and (5), since precession rate has been expressed in all three equations in units of fraction of cycle per one cycle. Inspection of planetary periods in relation to orbital eccentricities shows that T is independent of e and so actually period is NOT a function of eccentricity.

In view of the constant unit magnitude of T, the condition of mathematical equivalence of the two theories now can be expressed thus:

$$\pi = \frac{3}{1-e^2} \tag{8}$$

This approximate equality, Equation (8), is valid for a restricted range of orbital eccentricity values:

$$0 \le e \le 0.25 \tag{9}$$

The orbital eccentricities of all nine planets of the solar system fall within the range specified by Equation (9). The eccentricity of Mercury's elliptical orbit amounts to 0.205 and approximately conforms most closely among the nine planets with the stated condition of mathematical equivalence of the temporal theory and the general relativity theory.

Equations (8) and (9) suggest a test between these two theories, each of which sustains its claim of mathematical agreement with observed measures of perihelion excess precession rates. If one wishes to test further the truth or error of the temporal and general relativity theories of perihelion excess motion rate, then one can do so by seeking and investigating actual cases of central force motion involving a massive particle moving with a mean orbital velocity of 30 miles/second or more in an elliptical trajectory with an eccentricity as larger than 0.25 as attainable toward 1.00. The predictions of the two theories for all such cases significantly and measurably diverge. Whichever of the two theories gave less agreement with measured perihelion rates could be suspected of being wrong. Further tests would be required to learn whether the other theory is altogether right.

Appendix 1. Meaning of Coordinate Time in Reciprocal Space-Time

Mercury's perihelion excess motion is entirely due to its coordinate time, which should be added to its clock time to give the total time of the motion. The measure of Mercury's perihelion excess motion coordinate time is the ratio $V^2/c^2 = 2.55 \times 10^{-8}$. V = 29.8 miles/second denotes the mean orbital velocity of the planet Mercury. C = 186,281 miles/second denotes the outward equable rate of the scalar space-time progression. The value of the ratio is a scalar magnitude, since time has no spatial dimensions. The increase of coordinate time is radially outward from the radially inward gravitational motion of the planet Mercury. The resulting increase of spatial dislocation for Mercury's perihelion, equivalent to the measured coordinate time increase, is circumferential and so is measured through multiplying the coordinate time by π. Hence the circumferential space increase of the perihelion amounts to 8.02×10^{-8} in the unit of fraction of a cycle. Since there are 1.296×10^{6} seconds of arc length in one cycle, the Mercury perihelion excess motion amounts to 0.1039 seconds of arc length (equivalent to about 20 miles) per cycle.

To convert from units of fraction of a cycle per cycle to the customary units of seconds of arc length per century proceed as follows:

$$\frac{fraction\,of\;cycle}{1\,cycle} = fraction\,of\;cycle\;\frac{360\,°\,\frac{60\,'}{1\,°}\,\frac{60\,''}{1\,'}}{cycle}\;\frac{100\,years}{1\,century\;T\,\frac{years}{cycle}}$$
$$\frac{fraction\,of\;cycle}{cycle} = fraction\,of\;cycle\;\frac{1.296\times10^{8}}{T\;years}\;\frac{seconds\,of\;arc}{century} \tag{10}$$

Mercury's period of 0.2408 year divided into (8.02×10^{-8}) (1.296×10^{6}) = 0.1039 seconds of arc, multiplied by 100 gives 43.15 seconds of arc length per century for Mercury's perihelion excess motion. This is in fair agreement with Duncombe's measured value of 43.11 ± 0.45 seconds of arc length per century.

If the reader seeks a further exposition of the meaning of coordinate time, the reader would best go to the source of the concept which the author has found in the books of Larson[(2,7)].

The Larson view of the speed c as a scalar uniform unit rate of the space-time progression has the novel implication that vibrating photons of light remain in the space-time locations in which they originate and that the locations progress "with the speed of light." This view entertains the possibility, alluded to by Minkowski[8] in his well-known discussion about space and time, "whether space, which is supposed to be stationary, may not be after all in a state of uniform translation."

The Larson view that space and time are not infinitely divisible (not continua) also contradicts the Newton-Einstein conceptual heritage about the nature of space and time. I say to this that the infinite divisibility of space and time is not a proved truth. Santayana[9] has given material evidence for disallowing that the infinite divisibility of physical space and time is a necessary truth. Feynman[10] has stated: "I believe that the theory that space is continuous is wrong."

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8 Minkowski, H. *Space and Time*, a translation of an address delivered at the 80th Assembly of German Natural Scientists and Physicians, at Cologne, 21 September, 1908 in *The Principle of Relativity*, Dover, (Methuen & Co. 1923).

9 Santayana, G. *The Realm of Truth*, Charles Scribner's Sons, New York, 1938.

10 Feynman, R. *The Character of Physical Law*, The MIT Press, Cambridge, 1967.

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Appendix 2. Implicit Dependence of Perihelion Precession Rate on Orbital Eccentricity in both the Temporal Theory and the General Relativity Theory of Precession Rate

The crucial difference between the general relativity theory and the temporal theory of perihelion excess rate of precession is that there is an explicit dependence of this rate R on orbital eccentricity e, asserted by the relativity theory and denied by the temporal theory. The general relativity Equation 75 for R says further that the precession rate increases without bound as the eccentricity e approaches unity; the temporal theory rejects this statement of the behavior relation between R and e.

Besides committing itself to an explicit dependence of R on e, the general relativity theory is committed also to another, indirect and implicit dependence of R on e, but the latter dependence does not necessarily lead to an unbounded increase of R as e approaches unity. In this regard the temporal theory is not different from the relativity theory; the temporal theory is committed to the same implicit dependence of R on e as is the general relativity theory.

That the implicit dependence of R on e, contained both in the general relativity theory and the temporal theory of perihelion excess precession rate, does not necessarily lead to unbounded increase of R as e approaches unity, can be shown as follows:

The eccentricity e of an ellipse is given by

$$e=\sqrt{\frac{a^2-b^2}{a}} \qquad (11)$$

where a and b denote the semi major and semi minor axes respectively.

We have for each constant finite a that:

$$\lim_{b\to 0} e=\lim_{b\to 0}\sqrt{1-\frac{b^2}{a^2}}=1 \qquad (12)$$

We have for each constant finite b that:

$$\lim_{a\to \infty} e=\lim_{a\to \infty}\sqrt{1-\frac{b^2}{a^2}}=1 \qquad (13)$$

Thus, e can be regarded as approaching unity either by way of the semi-minor axis b approaching zero or by the way of the semi-major axis a approaching infinity.

The *implicit* dependence of perihelion precession rate R on e, manifest in both the general relativity theory and the temporal theory, derives from the assertion in both theories of a dependence of R on the square of a planet's orbital velocity V. Since V depends on a, as stated in Equation (3), and since e depends on a, as stated in Equation (11), it follows that R in this way implicitly depends on e, according to the temporal theory as well as to the general relativity theory.

The implicit dependence of perihelion precession rate R on e, however, does not necessarily lead to an unbounded increase of R as e approaches unity either for the general relativity theory or for the temporal theory.

The implicit dependence of R on e would lead to an unbounded increase of R as e approaches unity for both theories, if e could approach unity only through an unbounded increase of a, the semi-major axis. R would become unbounded for this case, because V would increase without bound. In this situation both the general relativity theory and the temporal theory would appear untenable, since infinite speed is improbable in the actual physical universe. Consequently, this avenue appears closed.

There is another avenue open, where we do not get these infinities and other difficulties for the implicit dependence of R on e.

The implicit dependence of R on e does not lead to an unbounded increase of R as e approaches unity for the temporal theory or for the general relativity theory, when e approaches unity by the finite decrease of the semi minor axis b toward zero. R remains bounded and finite, since V remains bounded and finite and V remains bounded and finite, because the semi major axis a now remains finite and bounded.